

Entangling unstable optically active matter qubits

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In distributed quantum computation, small devices composed of a single or a few qubits are networked together to achieve a scalable machine. Typically there is an optically active matter qubit at each node, so that photons are exploited to achieve remote entanglement. However, in many systems the optically active states are unstable or poorly defined. We report a scheme to perform a high-fidelity entanglement operation even given severe instability. The protocol exploits the existence of optically excited states for phase acquisition without actually exciting those states; it functions with or without cavities and does not require number resolving detectors.

One promising approach to quantum information processing (QIP) is *distributed* QIP [1–7]. Here scalability is achieved by networking many elementary nodes. Qubits in remote nodes are coupled through an entanglement operation (EO) which typically utilizes some kind of photon interference effect. Most of the proposed EOs are implemented not deterministically but rather are probabilistic in nature. Failure of the EO will be ‘heralded’, i.e. the operator is aware of the failure, but in that case the qubits acted upon are effectively corrupt and need to be reset. Therefore if only one qubit is present at each node, performing an EO between two specific qubits implies a significant risk of losing any prior entanglement with other qubits.

There are many existing suggestions for implementing EOs, including a number of so-called path-erasure schemes [1–5, 11, 12, 15]. These approaches typically involve exciting an optical transition in the matter system at each node. However, in many real systems such transitions are inherently unstable due to energy fluctuation of the excited states. If such states exist in superposition with lower lying states, even briefly, then their instability will cause dephasing and hence ultimately degradation of the entanglement operation [16, 17]. So one has to look for a robust way to generate high-fidelity entanglement under the effect of such energy fluctuations.

Recently it has been shown that one can suppress such dephasing through temporal postselection of emitted photons at the expense of decreasing success probability [18]. However, this scheme is sensitive to imperfections in the photodetector. Moreover, due to the inherently low success probability, dark counts will be a relevant problem and will lead to decreased fidelity [19]. Here we present a fully scalable procedure for distributed quantum computation by constructing a high-fidelity EO scheme which is relatively robust against such issues.

We begin by describing a simplified scenario and then consider realistic imperfections. We will use the term ‘atom’ to refer to a generic optically active qubit, which may in fact be a quantum dot or crystal defect (we discuss such possibilities presently). In essence, a single photon detuned from the

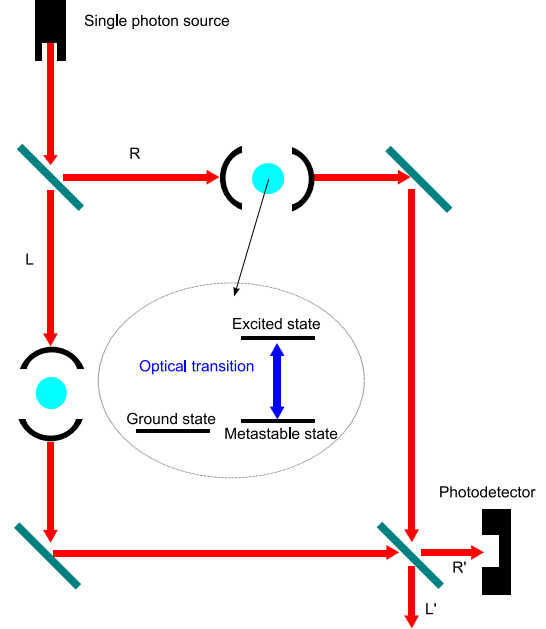


FIG. 1: Schematic of an apparatus for the EO. A half mirror splits a single photon (or a weak coherent laser) into two paths. A matter qubit (denoted ‘atom’, but maybe quantum dot etc) is confined in a cavity on each path. Each atom has an L-type structure and has three quantum states $|0\rangle$, $|1\rangle$, and $|e\rangle$ as shown in this schematic. The photon frequency matches a cavity mode and so the photon transmission probability at the cavity can approach unity. Ultimately the photon goes through another splitter and, depending on output port, may be measured by a photon detector. A detection event projects the atoms into an entangled state. Since there is a large detuning between a cavity frequency and the atomic transition energy, the optical transition is suppressed, and therefore the excited state can have a large energy fluctuation without decreasing the fidelity of the entanglement.

atomic transition induces a relative phase between the atomic states. Since the atom(s) never undergo a transition to the optically excited state, the scheme can be extremely robust against fluctuations in such states. Surprisingly, even when the envi-

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ronmental coupling strength is the same order of magnitude as the atom-photon coupling strength, one can still generate a high-fidelity entanglement with a reasonable success probability.

We assume that a matter qubit in a cavity has an L-type structure and has three quantum states $|0\rangle$, $|1\rangle$, and $|e\rangle$. A state $|1\rangle$ is optically active and coupled to a noisy excited state $|e\rangle$ whereas the state $|0\rangle$ is not optically active (see Fig. 1). Two such atoms are remotely located, each within a cavity (we discuss the case without cavities presently). Initially the atoms are prepared as $|+\rangle_L|+\rangle_R = \frac{1}{2}(|0\rangle_L|0\rangle_R + |0\rangle_L|1\rangle_R + |1\rangle_L|0\rangle_R + |1\rangle_L|1\rangle_R)$ where L and R denote the location of each cavity. When a frequency of a cavity mode is detuned from an atomic transition, a simplistic effective interaction Hamiltonian can be written as [20]

$$H_{\text{eff}} \simeq \frac{g^2}{\Delta} (|e\rangle\langle e| - |1\rangle\langle 1|) \hat{a}^\dagger \hat{a} \quad (1)$$

where g is the coupling strength of the cavity and Δ is the detuning between the cavity mode and the atomic transition. This effective Hamiltonian can be derived from the standard Jaynes-Cummings Hamiltonian in the limit of large detuning.

We suppose that a single photon is split by a half mirror into two paths, along which the two cavities lie symmetrically. Importantly the photons are of a frequency that is significantly detuned from the atomic transitions; the cavity mode frequencies are matched to the frequency of the photons. Despite the detuning, when a photon interacts with an atom in the optically active atomic state $|1\rangle$ a certain phase is acquired. However there is no interaction between the photon and the atom for an optically inactive state $|0\rangle$.

Subsequent to the atom-photon interaction, our state $|+\rangle_L|+\rangle_R$ has evolved to

$$\frac{1}{2\sqrt{2}} \left(|0\rangle_L|0\rangle_R (\hat{a}_L^\dagger + \hat{a}_R^\dagger) + |0\rangle_L|1\rangle_R (\hat{a}_L^\dagger + \hat{a}_R^\dagger e^{i\theta}) + |1\rangle_L|0\rangle_R (\hat{a}_L^\dagger e^{i\theta} + \hat{a}_R^\dagger) + |1\rangle_L|1\rangle_R (\hat{a}_L^\dagger e^{i\theta} + \hat{a}_R^\dagger e^{i\theta}) \right) \quad (2)$$

where \hat{a} (\hat{a}^\dagger) denote an annihilation (creation) operator of the photon. The phase is effectively described as $\theta \simeq \frac{g^2}{\Delta} t$ where t is an interaction time. Since the frequency of the photon is assumed to be centered around the cavity frequency in a narrow range, the photon can be transmitted through the cavity without reflection [21]. Finally the photon goes through another half mirror to change the mode \hat{a}_L^\dagger (\hat{a}_R^\dagger) into $\hat{a}_{L'}^\dagger = \frac{\hat{a}_L^\dagger + \hat{a}_R^\dagger}{\sqrt{2}}$ ($\hat{a}_{R'}^\dagger = \frac{\hat{a}_L^\dagger - \hat{a}_R^\dagger}{\sqrt{2}}$) where L' and R' denote the output ports of the splitter, and we obtain

$$\begin{aligned} & \frac{e^{i\theta}}{2} |0\rangle_L|0\rangle_R \hat{a}_{L'}^\dagger + |0\rangle_L|1\rangle_R \frac{e^{\frac{1}{2}i\theta}}{2} (\cos \frac{\theta}{2} \hat{a}_{L'}^\dagger + i \sin \frac{\theta}{2} \hat{a}_{R'}^\dagger) + \\ & \frac{e^{\frac{1}{2}i\theta}}{2} |1\rangle_L|0\rangle_R (\cos \frac{\theta}{2} \hat{a}_{L'}^\dagger - i \sin \frac{\theta}{2} \hat{a}_{R'}^\dagger) + |1\rangle_L|1\rangle_R \hat{a}_{L'}^\dagger. \end{aligned} \quad (3)$$

If the atoms were not present, or were in either the definite state $|0\rangle_L|0\rangle_L$, or the state $|1\rangle_L|1\rangle_L$, then the photon

would certainly exit from the left port of the second splitter. However, because of the internal phase shifts, the photon may exit from the right port – this represents a successful entanglement. The success probability is $\frac{1}{2} \sin^2 \frac{\theta}{2}$ which provides us with a maximum value 0.5 for $\theta = \pi$, and regardless of θ this measurement projects the atomic state into an entangled state $\frac{1}{\sqrt{2}}|0\rangle_L|1\rangle_R - \frac{1}{\sqrt{2}}|1\rangle_L|0\rangle_R$. In this example the initial state was assumed to be $|+\rangle_L|+\rangle_R$, however one can perform this operation on arbitrary initial states, and success results in a parity projection i.e. a projector onto a odd parity two-qubit subspace, which is one of the typical EOs for distributed quantum computation [2, 4, 7, 15].

In practice no single photon source will be ideal. Therefore we now consider both an imperfect single photon source, and a weak coherent laser, as alternatives. The ideal single photon source should emit one and only one photon when the device is triggered, which can be realized to some approximation by exploiting the photon antibunching effect [22]. With current technology it is inevitable that the pulse generated by a source may contain either no photons, or multiple photons, with finite probability. Suppose that P_m denotes the probability of emitting m photons. Importantly, finite P_0 will not give rise to errors; in effect it adds to the probability of photon loss and will be registered as a failure by the detectors. However, finite P_m with $m \geq 2$ will give rise to errors; for example, for $m = 2$ a likely occurrence is that one photon will be lost and the other registered by the detector, in which case we would wrongly conclude that the normal EO has succeeded. Taking the worst case that such emissions can make the state orthogonal to the target state, the fidelity is bounded as $F \geq 1 - \sum_{m \geq 2} P_m$. Fortunately, it is possible to make the probability of such events rather small. For example, Ref. [23] reports a single photon source whose P_0 and P_2 are 14% and 0.08% respectively (with negligible chance of higher m).

For near future demonstrations of the protocol described here, it may be appropriate to utilize an even less ideal source: namely a weak coherent laser. A coherent state is described as $|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{1}{n!} \alpha^n |n\rangle$ where $|\alpha|^2$ denotes a mean number of the photons and $|n\rangle$ denote a number state of the photon. A coherent state $|\alpha\rangle$ acquires a phase as $|\alpha e^{i\theta}\rangle$ when the atomic state is optically active. So, taking the same operation as a single photon mentioned above, the state following the final beam-splitter will be $\frac{1}{2}|\alpha\rangle_{L'}|\text{vac}\rangle_{R'}|0\rangle_L|0\rangle_R + \frac{1}{2}|\alpha e^{\frac{i\theta}{2}} \cos \frac{\theta}{2}\rangle_{L'}|\alpha e^{\frac{i\theta}{2}} \sin \frac{\theta}{2}\rangle_{R'}|0\rangle_L|1\rangle_R + \frac{1}{2}|\alpha e^{\frac{i\theta}{2}} \cos \frac{\theta}{2}\rangle_{L'} - \alpha e^{\frac{i\theta}{2}} \sin \frac{\theta}{2}\rangle_{R'}|1\rangle_L|0\rangle_R + \frac{1}{2}|\alpha\rangle_{L'}|\text{vac}\rangle_{R'}|1\rangle_L|1\rangle_R$ where $|\text{vac}\rangle_{R'}$ denotes a vacuum state of mode R' . Detecting a single photon of the mode R' projects the atomic state into an entangled state $|\psi^{(-)}\rangle = \frac{1}{\sqrt{2}}|0\rangle_L|1\rangle_R - \frac{1}{\sqrt{2}}|1\rangle_L|0\rangle_R$, while detecting two photons project the state into $|\psi^{(+)}\rangle = \frac{1}{\sqrt{2}}|0\rangle_L|1\rangle_R + \frac{1}{\sqrt{2}}|1\rangle_L|0\rangle_R$. So a photon number resolution device can project the atomic state into a Bell state. However, if one wished to operate the protocol without the availability of a reliable photon number resolution device, one could still obtain entanglement between the matter qubits. In that case when the detector at the output R' registers (a non-

zero but unknown number of) photons, the state of the matter qubit becomes a classical mixture of the target state $|\psi^{(-)}\rangle$ and the other error states $|\psi^{(+)}\rangle$. A fidelity F and a success probability P are scaled as $F = 1 - \frac{1}{2}|\alpha|^2 \sin^2 \frac{\theta}{2} + O(|\alpha|^4)$ and $P = \frac{1}{2}|\alpha|^2 \sin^2 \frac{\theta}{2} + O(|\alpha|^4)$ for $|\alpha|^2 \ll 1$, respectively. Here, at the expense of success probability, a weaker coherent state can increase the fidelity by guaranteeing that it is unlikely for multiple photons to reach the detector. So one can obtain high fidelity entanglement by using a weak coherent state. Although the success probability becomes relatively low due to the trade-off relationship between the fidelity and the success probability, a coherent laser is much easier to construct than a single photon source, and so this scheme using a coherent state should be feasible even with current technology.

Photon loss is a major source of error in the most of previously proposed EOs [1, 3–6, 12], including the ingenious schemes [12–14] which, like the present scheme, operate by inducing a phase. In effect we propose to exploit the fact that single photon sources are becoming a mature technology and can therefore be substituted for the classical source in that previous scheme, with the benefit that photon loss will now be detected and thus prevented from impairing fidelity. Also, it is worth mentioning that cavities are not essential for our scheme as long as one can achieve a strong coupling between a photon and an atom. For example, recently, strong interaction between light and a single atom in a free space has been demonstrated by using a lens [24], and a phase shift of a weak coherent state about 1° , which is induced by a single atom, has been observed experimentally [25]. Since our analysis above can be directly applied to the case of free space, these experiments also demonstrate the feasibility of performing our scheme without the need for a cavity.

We now present a more detailed analysis. In the previous description we adopted an approximation that there is no optical transition because of a large detuning between the atomic transition and the frequency of the photon. However, even when the detuning Δ is large, there is a non-zero probability for the state to be excited, which might affect the fidelity of the EO. To include this effect, we use the following Hamiltonian instead of the effective Hamiltonian (1)

$$H = \sum_{j=L,R} \left(\frac{\omega}{2} \hat{\sigma}_z^{(j)} + \nu \hat{a}_j^\dagger \hat{a}_j \right) + \sum_{j=L,R} g \left(\hat{\sigma}_+^{(j)} \hat{a}_j + \hat{\sigma}_-^{(j)} \hat{a}_j^\dagger \right) - i\Gamma (\hat{a}_L^\dagger \hat{a}_L + \hat{a}_R^\dagger \hat{a}_R) \quad (4)$$

where ω , ν , g , and Γ denote the atomic transition energy, the cavity frequency, the coupling strength of the cavity, and the decay rate of the cavity respectively. Note that we have added not only the standard Jaynes-Cummings Hamiltonian (the first and second term) but also a decay term (the last term) to describe the conditional dynamics when no photon is measured at the detector [26]. Importantly, $\frac{1}{\Gamma}$ gives the characteristic time during which the photon in the cavity interacts with the atom. For a large detuning, the effective Hamiltonian (1) is

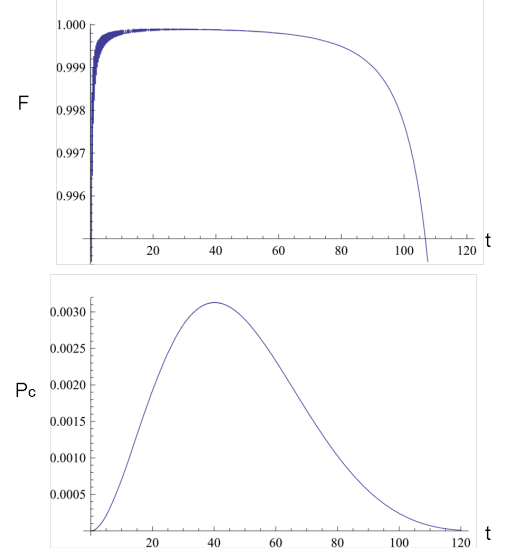


FIG. 2: Upper: The fidelity of an EO using a single photon source is plotted, where t denotes the time of the detection by the photon detector. Lower: $P_c(t)$, the probability of the desired detector ‘click’ per unit time. The average fidelity is calculated as 0.998 and the total success probability is 0.178. We set parameters as $\Delta = 20g$, $\Gamma = \frac{1}{\pi} \frac{g^2}{\Delta}$, $\lambda = 0.1g$, and $g = 1$ where Δ , Γ , λ , and g denote a detuning, a cavity decay rate, an environmental coupling, and the atom-cavity coupling respectively.

a good approximation, and therefore it is necessary to satisfy $\frac{g^2}{\Delta} \frac{1}{\Gamma} \simeq \pi$ so that one can obtain a reasonable success probability. To satisfy this requirement, a strong coupling regime $g \gg \Gamma$ is in turn required since we will employ a Δ which is much larger than the coupling strength g (to prevent the state from being excited).

To model the effect of the energy fluctuation of the excited state, we use a Lindblad master equation as follows:

$$\frac{d\rho}{dt} = -i(H\rho - \rho H^\dagger) - \lambda \sum_{j=L,R} [|e\rangle_j \langle e|, [|e\rangle_j \langle e|, \rho]]. \quad (5)$$

The solution of this master equation provides us with a density matrix of the state while the detector registers no photons. When the detector registers an event in mode R' observed at time t , the state is projected onto $\rho_{\text{final}}(t) = \hat{a}_{R'} \rho(t) \hat{a}_{R'}^\dagger$ discontinuously. For $\Delta = 20g$, $\Gamma = \frac{1}{\pi} \frac{g^2}{\Delta}$, and $\lambda = 0.1g$, we have plotted fidelity $F(t) = \langle \psi(t)_{\text{bell}} | \rho_{\text{final}} | \psi_{\text{bell}} \rangle$ in Fig. 2 where $|\psi_{\text{bell}}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. This figure shows that, after the fidelity takes a maximum value, it decays due to the energy fluctuation. By taking an average of the fidelity, we obtain $F_{\text{average}} = \int_0^\infty P_c(t) F(t) dt \simeq 0.998$ where $P_c(t)$ denotes a probability of clicking the photon at a time t (also shown in the Figure).

By integrating the success probability over the time t , we obtain a total success probability as 0.177 and this success probability is large enough to grow a resource such as a clus-

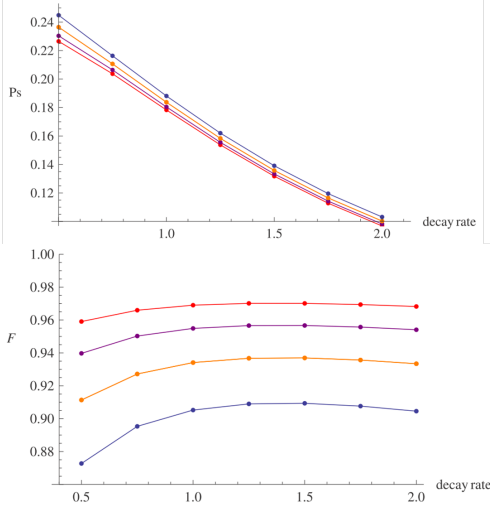


FIG. 3: The success probability and fidelity of an EO using a single photon source is plotted against the normalized decay rate of a cavity with $\gamma = \Gamma/(\frac{1}{\pi} \frac{g^2}{\Delta})$. The highest line (blue) in the upper graph and the lowest line (blue) in the below graph are the case of $\Delta = 15g$, and the other lines are the case of $\Delta = 20g, 25g, 30g$ (orange, purple, red), respectively. Also, we have fixed the environmental coupling and the atom-cavity coupling as $\lambda = g$.

ter state on a practical time scale [2, 8–10] without employing a brokering strategy[7]. Furthermore, we have studied how the decay rate changes the total success probability and the average fidelity. In Fig. 3, we have plotted the average fidelity and success probability against a normalized decay rate $\gamma = \Gamma/(\frac{1}{\pi} \frac{g^2}{\Delta})$ when the environmental coupling strength λ is the same value as the coupling strength g . The fidelity takes a maximum value when the normalized decay rate is around $\frac{3}{2}$. Even when the coupling strength of the noise is the same order of the magnitude as the atom-cavity coupling strength, our scheme can still generate entanglement with a reasonable success probability and good fidelity.

In Fig. 4, we show the case of using a weak coherent state. For $\Delta = 7g$, $\Gamma = \frac{1}{\pi} \frac{g^2}{\Delta}$, $\lambda = 0.5g$, and $\alpha = 0.2$, the average fidelity is 0.939 and the total success probability is 0.0128. While this success probability may be too low to support universal quantum computing (without the use of additional memory qubits [7]), it is however certainly high enough to permit smaller scale applications or a comprehensive experimental demonstration of the protocol.

Finally, we describe possible experimental realizations. A quantum dot (QD) defined on n-type GaAs heterostructures is one of the candidates. A polarized photon can selectively drive one of the electron spin states into an excited state called a trion, and so it is possible to construct the needed L-type structure. Moreover, strong coupling with a photon has already been realized in a QD in a semiconductor microcavity where $g = 80\mu\text{ev}$ and $\Gamma = 33\mu\text{ev}$ [17, 27]. Another candidate is a p-type GaAs QD where a single hole has two spin states. Importantly, a long spin relaxation time of the spin

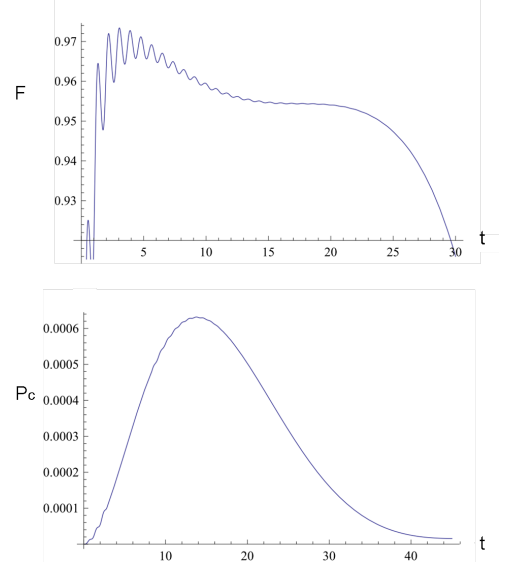


FIG. 4: Upper: The fidelity of an EO using a weak coherent state is plotted, where t denotes the time of the detection by the photon detector. Lower: $P_c(t)$, the probability of the desired detector ‘click’ per unit time. The total success probability is 0.0128 and the average fidelity is 0.939. We set parameters as $\Delta = 7g$, $\Gamma = \frac{1}{\pi} \frac{g^2}{\Delta}$, $\lambda = 0.5g$, $\alpha = 0.2$, and $g = 1$ where Δ , Γ , λ , α , and g denote detuning, decay rate of the cavity, environmental coupling, amplitude of the coherent state, and the atom-cavity coupling respectively.

$T_1 \simeq 1$ ms has been demonstrated [28], and the same order of spin dephasing time T_2 is theoretically predicted in the hole spin states [29], which is much longer than the electron spin dephasing time $T_2 \simeq 10$ ns in a n-type QD [30]. These otherwise attractive systems are impaired by the optical emission of the hole spin states which has a large line width implying the excited states are more noisy than n-doped QDs [28]. This is therefore a very relevant class of system for the present scheme, by which it is possible to perform high fidelity EOs despite the noisy excited state.

Nitrogen vacancy (NV) centers in diamond are a second system with promising properties (including an electron dephasing time about a millisecond at a room temperature [31]), marred by an optically excited state with strong phonon interactions. Therefore this system is also very relevant to the present scheme, although a suitable strong coupling with a photon through a cavity has not yet been demonstrated.

In conclusion, we have described a scheme to entangle distant matter qubits even when those qubits suffer severe energy fluctuations. Many optically active solid state systems suffer unstable excited states, and our scheme provides a practical way to overcome such typical issues. The authors thank J.M. Smith for a useful discussion. This research is supported by the National Research Foundation and Ministry of Education, Singapore. JF acknowledges support from Merton College. YM is supported by the Japanese Ministry of Education, Culture, Sports, Science and Technology.

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